Advanced Research
Intelligent
Embedded
Systems

Uncertainty, Information and Learning Mechanisms (Part 2)

Intelligence for Embedded Systems

Ph. D. and Master Course

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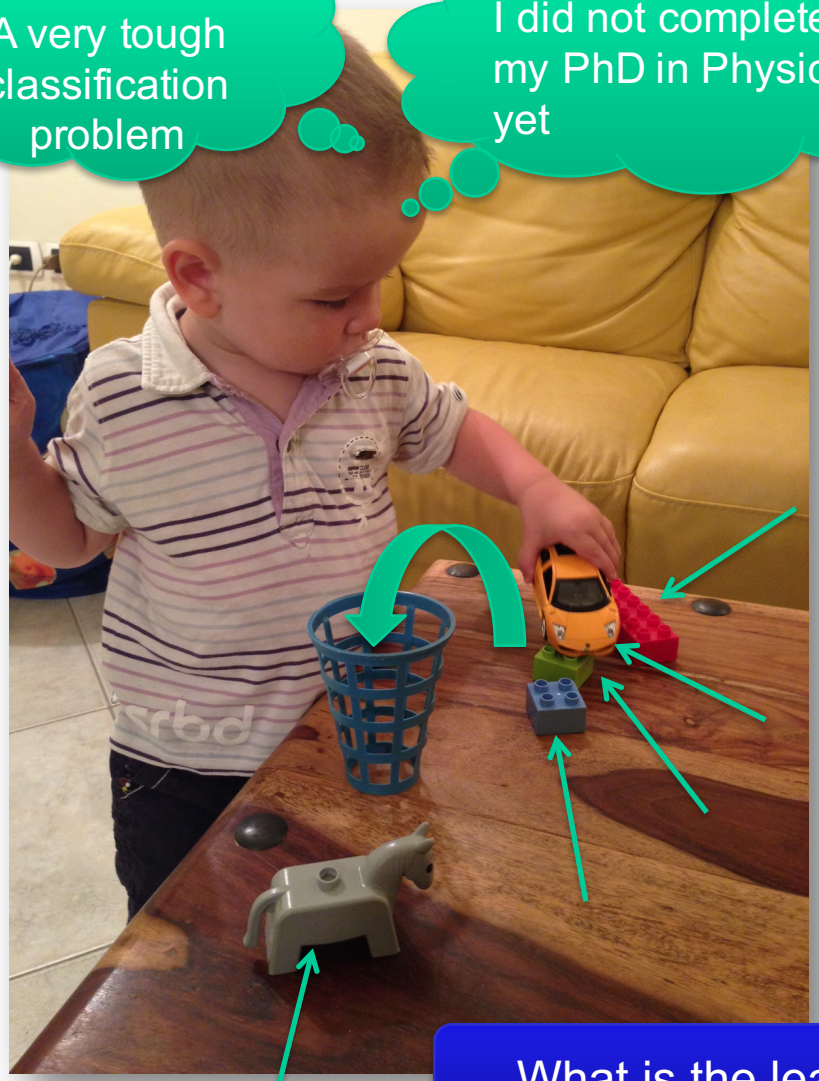
Uncertaintanty and Learning

- The real world is prone to **uncertainty**
- At different level of the data analysis process
  - Acquiring data
  - Representing information
  - Processing the information
  - Learning mechanisms
- **Two questions**:
  - ✓ Why do we need learning mechanisms?
  - ✓ Which are basics of supervised statistical learning?

First lecture

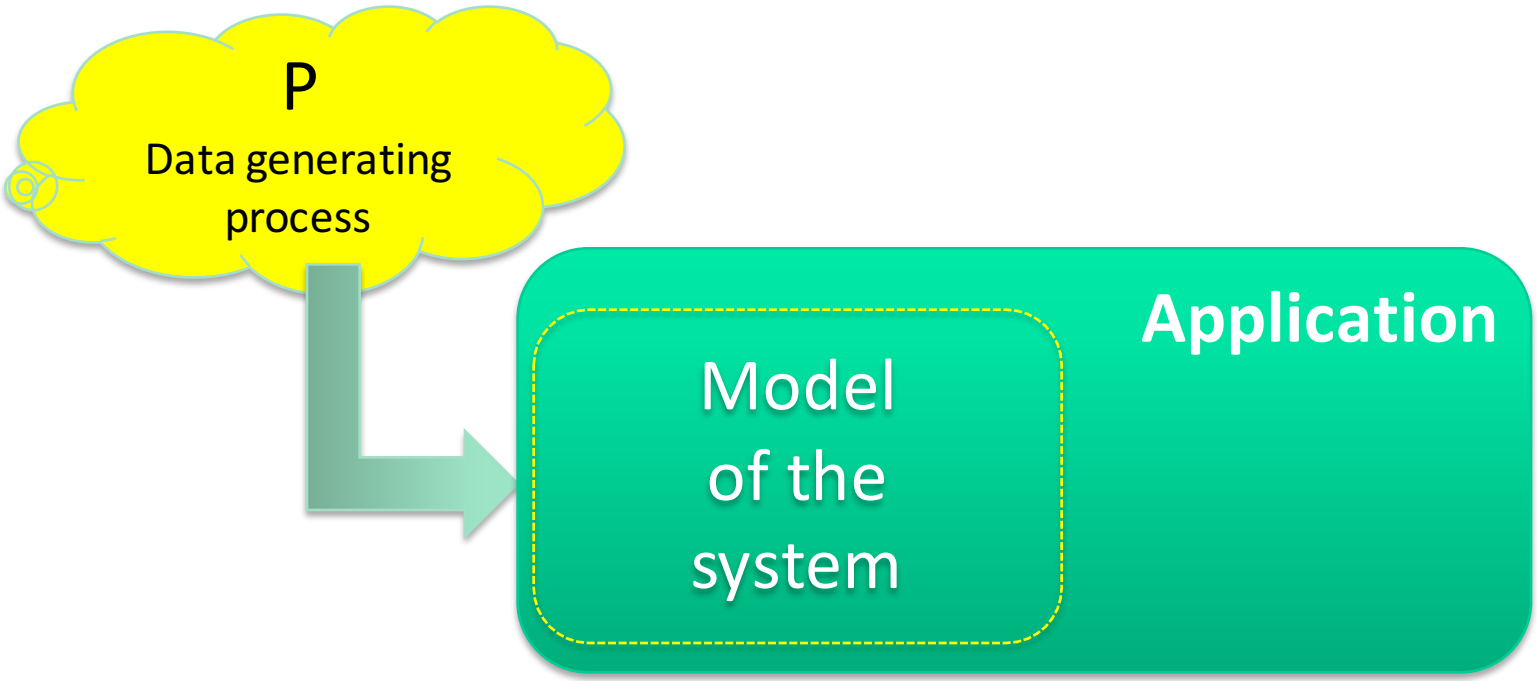
Part 1 of this lecture

A “toy” example



What is the learning goal here?

Data-processing and applications



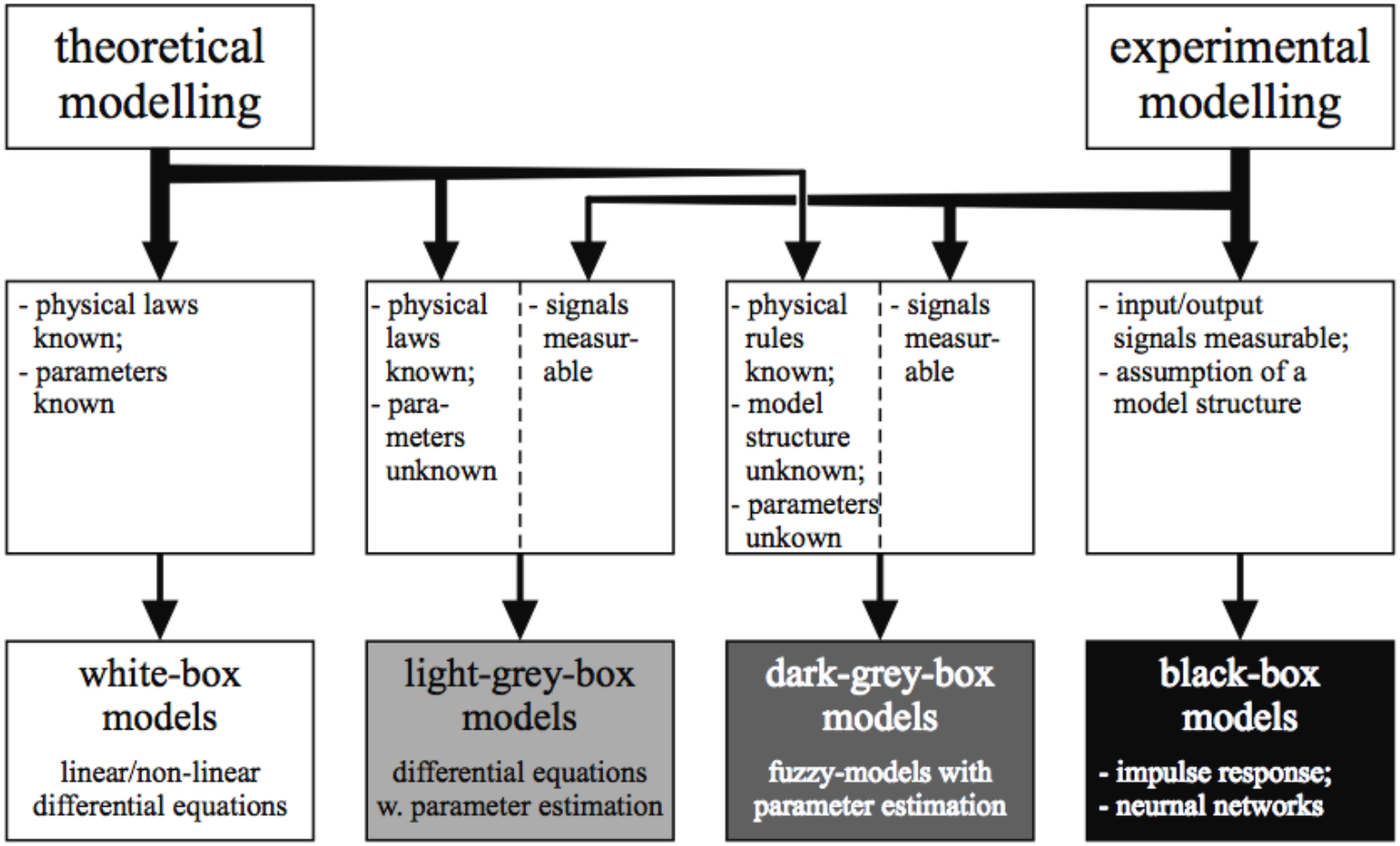
System modeling

Not rarely we wish to generate a functional dependency among sensor datastreams (model) to solve a problem. Some examples

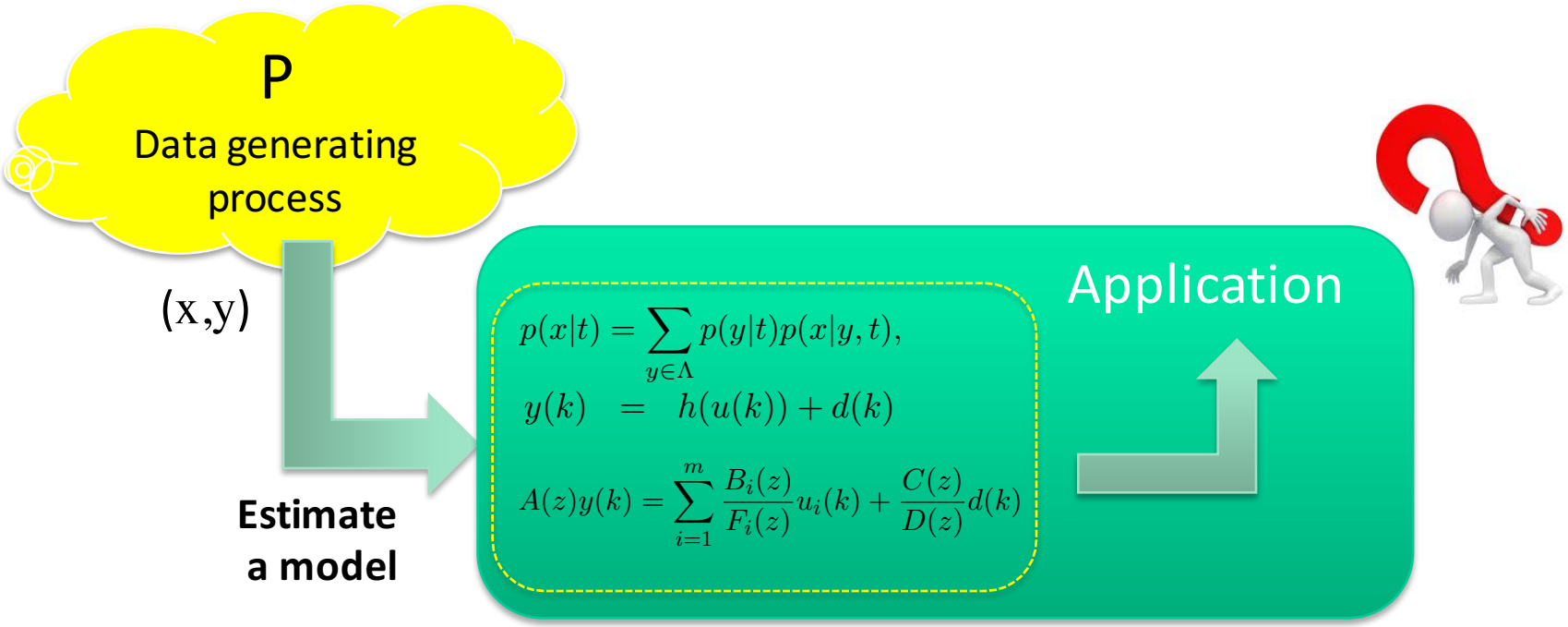
- **Design a Classifier**
  - e.g., optical character recognition, face recognition, explosive detection, quality analysis in the manufactoring industry…
- **Construct a functional dependency** (function regression)
  - e.g., function reconstruction, data regularization, predictive modeling…

System modeling

- In some cases the physical equations describing the process are available but some parameters need to be determined
- In others, we do not know the equations ruling the system
- If the considered model is linear we speak about system identification, when not linear about learning


theoretical modelling
experimental modelling
- physical laws known;
- parameters known
- physical laws known;
- para-meters unknown
- signals measur-able
- physical rules known;
- model structure unknown;
- parameters unkown
- signals measur-able
- input/output signals measurable;
- assumption of a model structure
white-box models
linear/non-linear differential equations
light-grey-box models
differential equations w. parameter estimation
dark-grey-box models
fuzzy-models with parameter estimation
black-box models
- impulse response;
- neurnal networks

Learning the system model

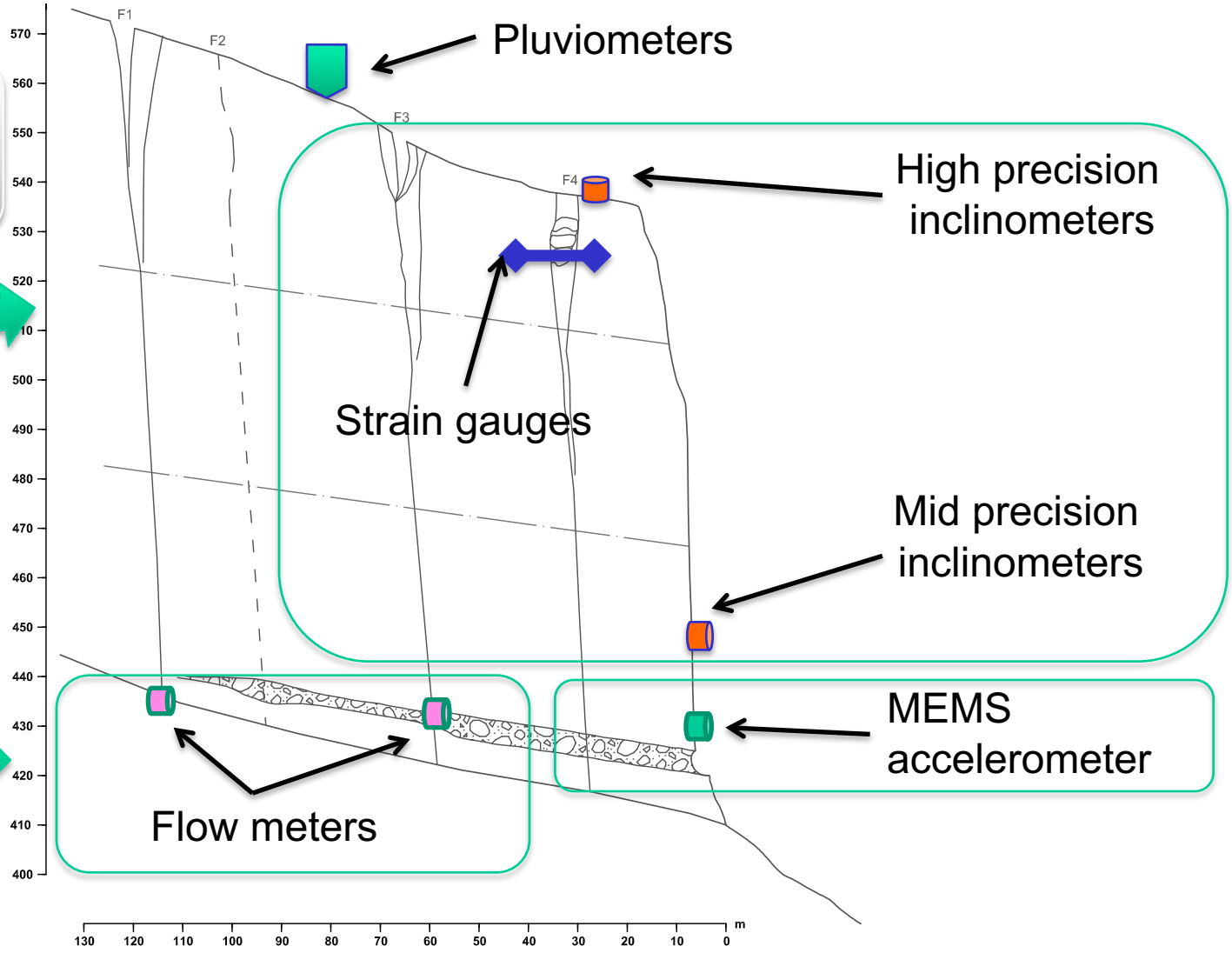


We will come back to the learning mechanism later

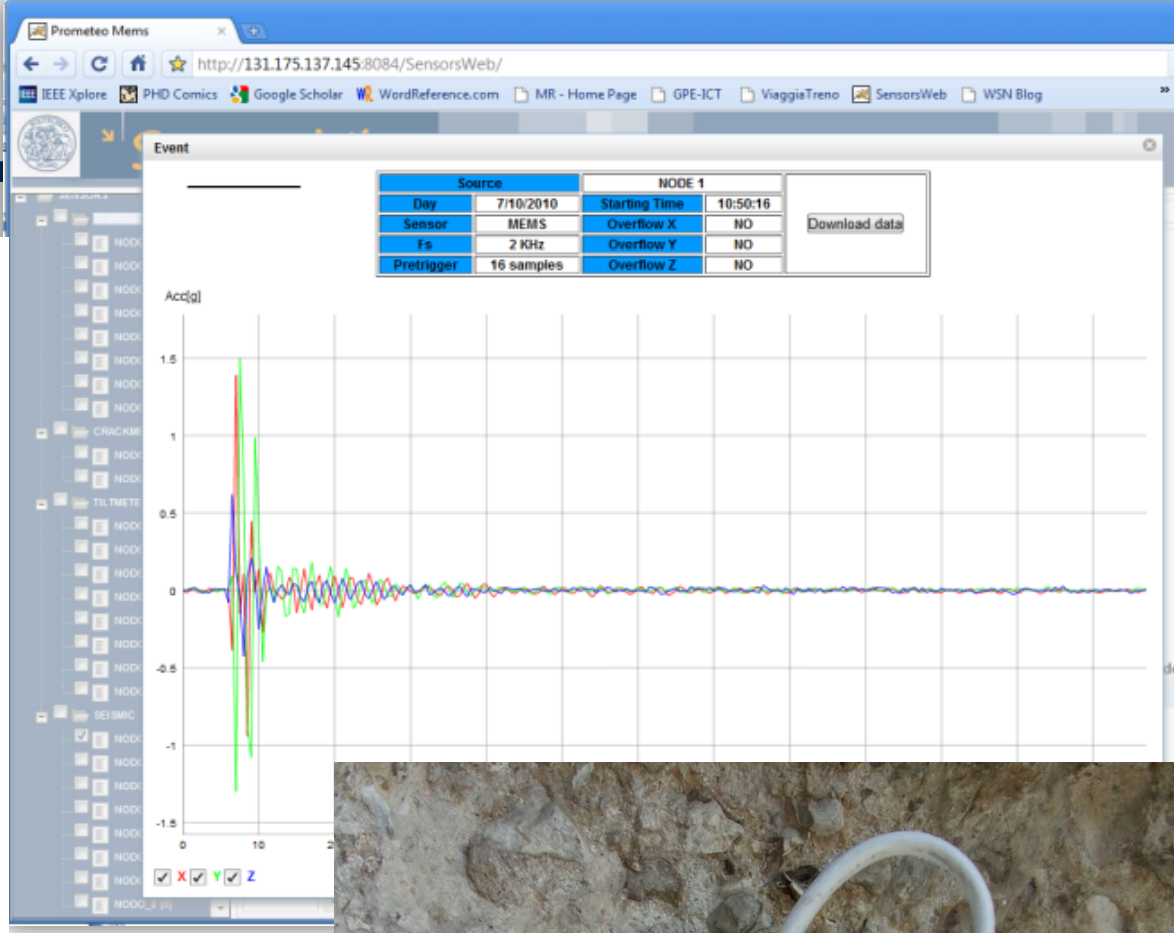
- **A real example: classifying microacoustic bursts**

The Torrioni di Rialba Monitoring system

In addition:
Many temperature sensors



- We wish to automatically identify microacoustic bursts to reduce the false alarms (data trigger)
- The application requires to design a **classifier** assigning a relevant/not relevant label to bursts



Designing a Classifier

1. Pre-Processing

2. Feature Extraction

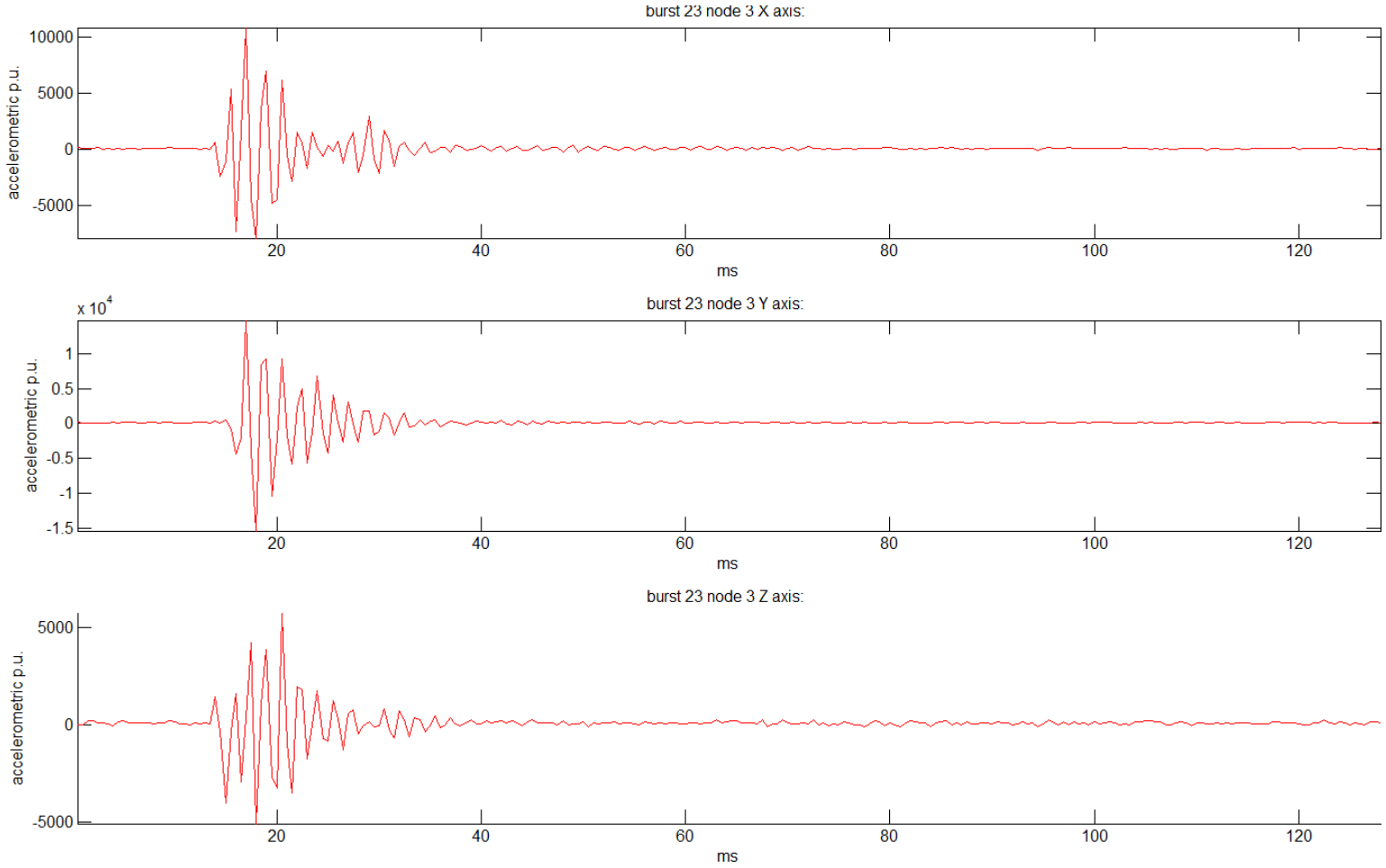
3. Design the Classifier

4. Use the classifier

- Burst alignment: bursts need to be aligned to provide a common comparison
- Bursts are then processed to extract scalar features
- Design the classifier from available bursts
- Bursts are automatically classified

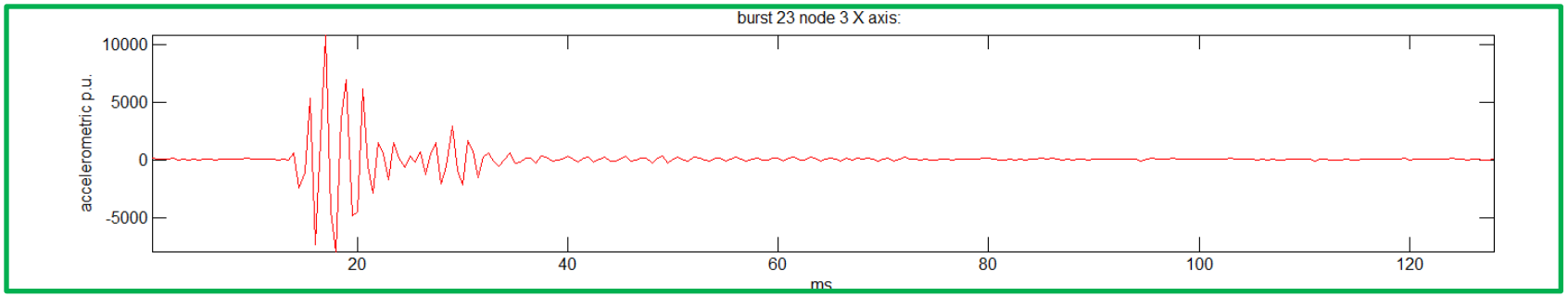
Microacoustic Bursts

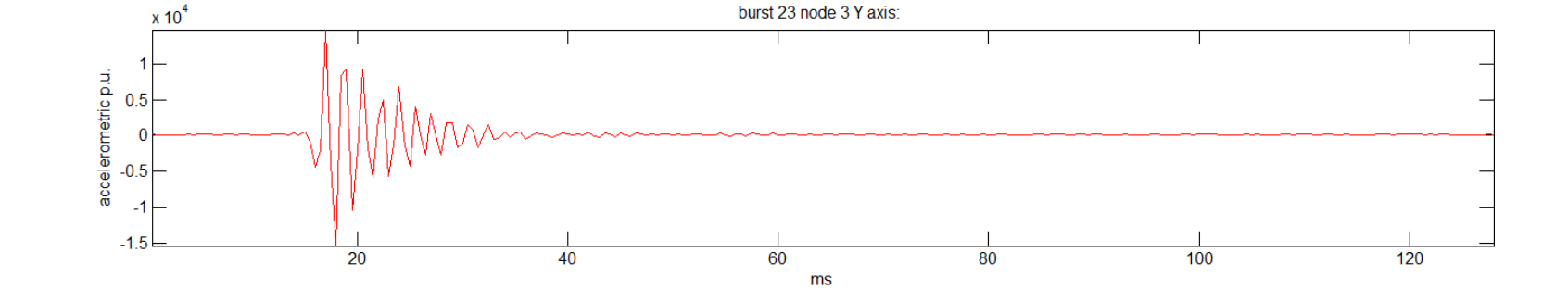
- Each burst is projected onto an adaptive orthonormal three-axis system estimated with a Principal Component Analysis (PCA)

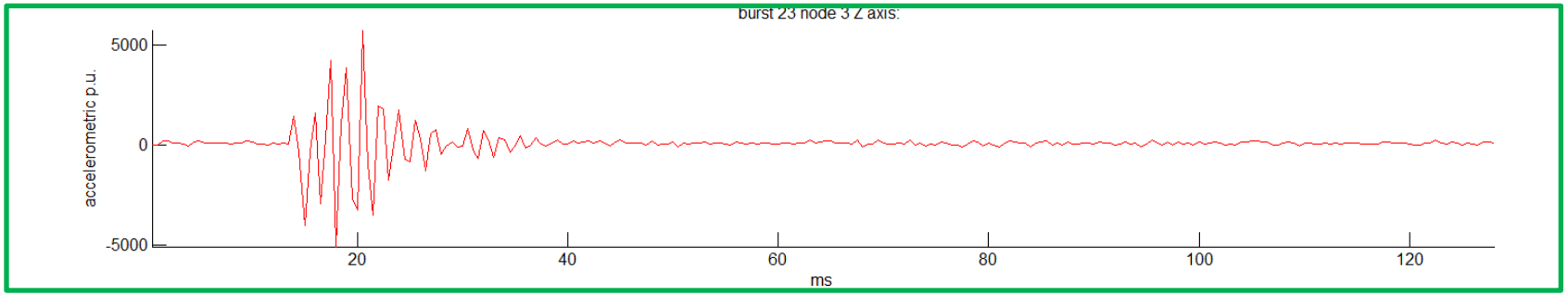


Pre-processing

- Only the principal and the leat signifcant components are further processed



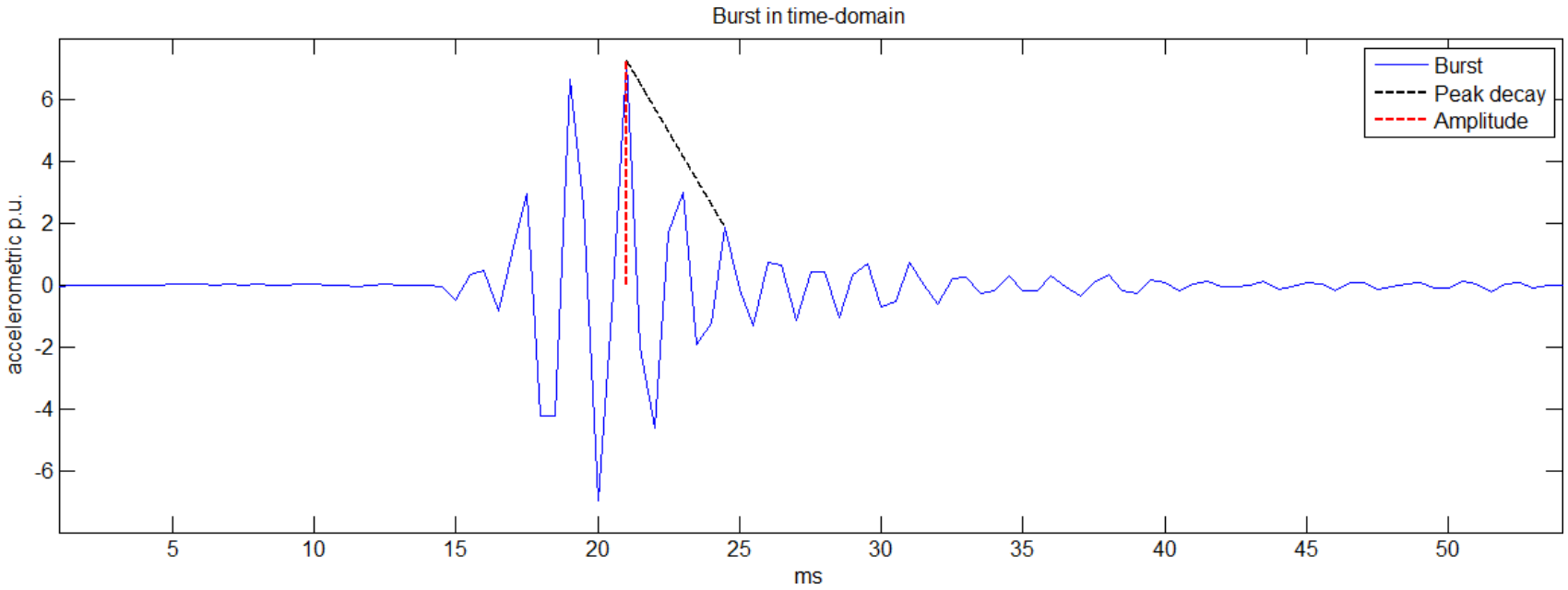




Feature Extraction

Time Domain Features

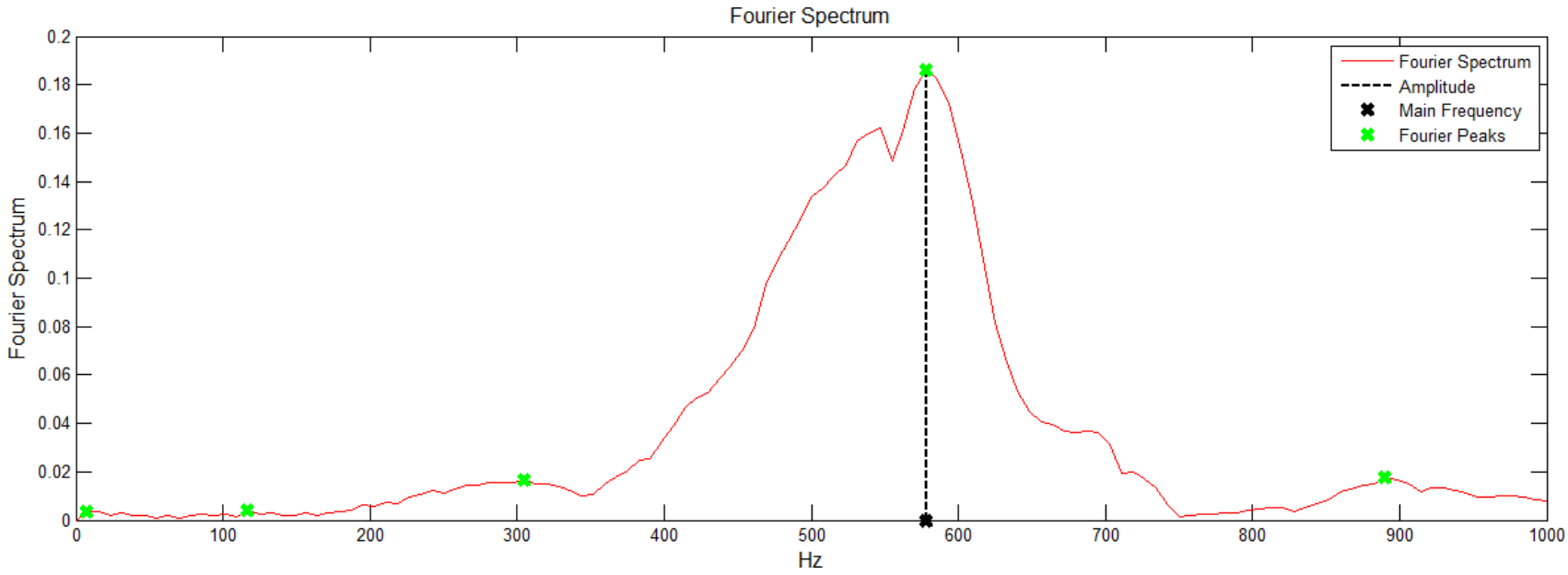
- Mean (before PCA)
- PCA eigenvalue
- Max amplitude
- SNR
- PSNR
- Peak decay



Feature Extraction

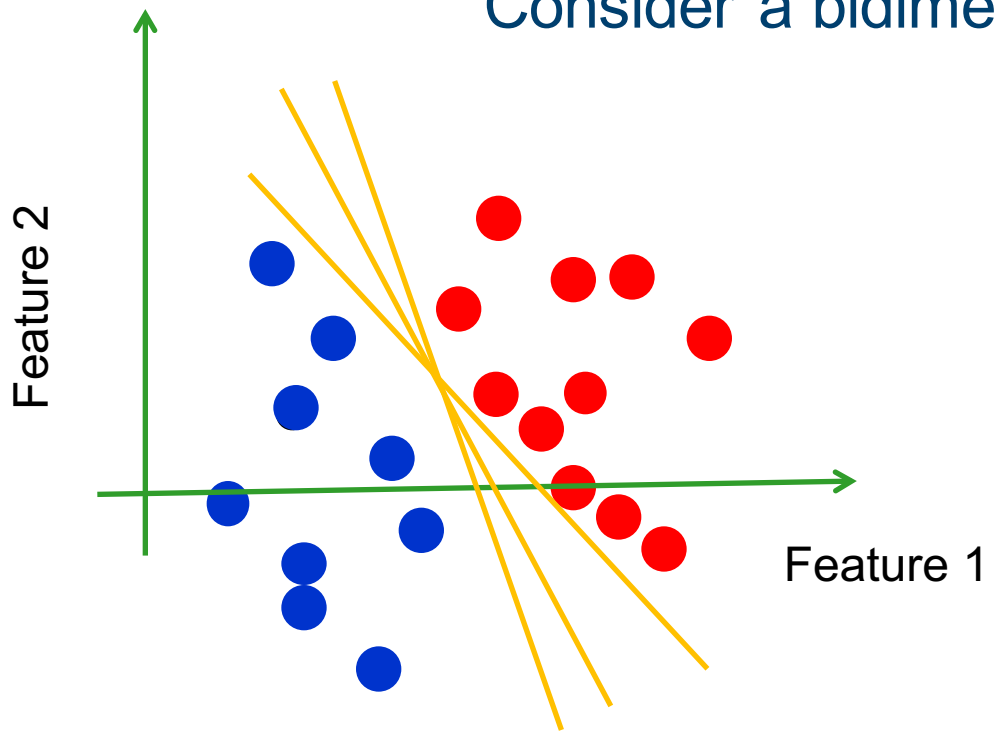
Fourier Domain Features (Power Spectrum)

- Max amplitude
- Main frequency
- Variance of peaks



Design a classifier

Consider a bidimensional problem

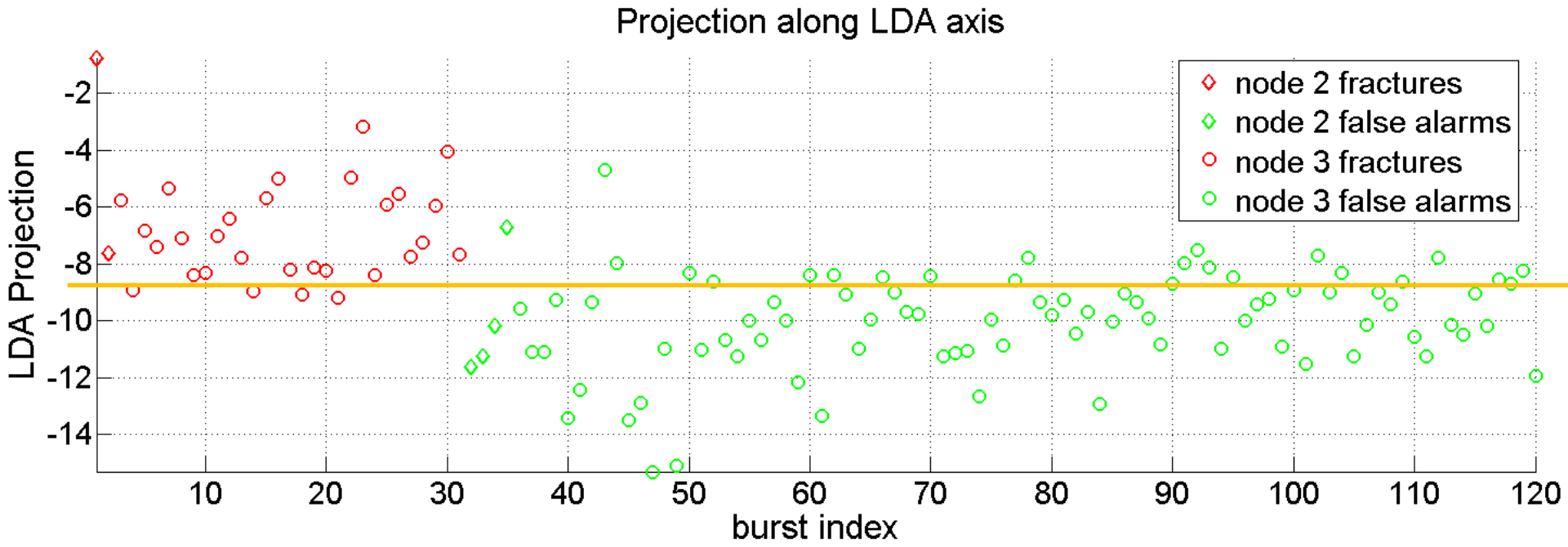


Designing a classifier requires identification of the function separating the labeled points

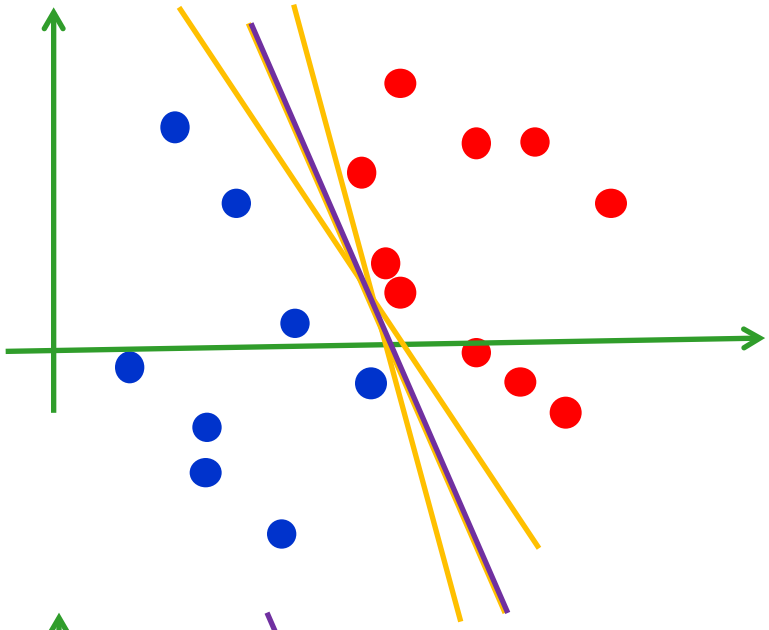
Designing a classifier: Burst separation

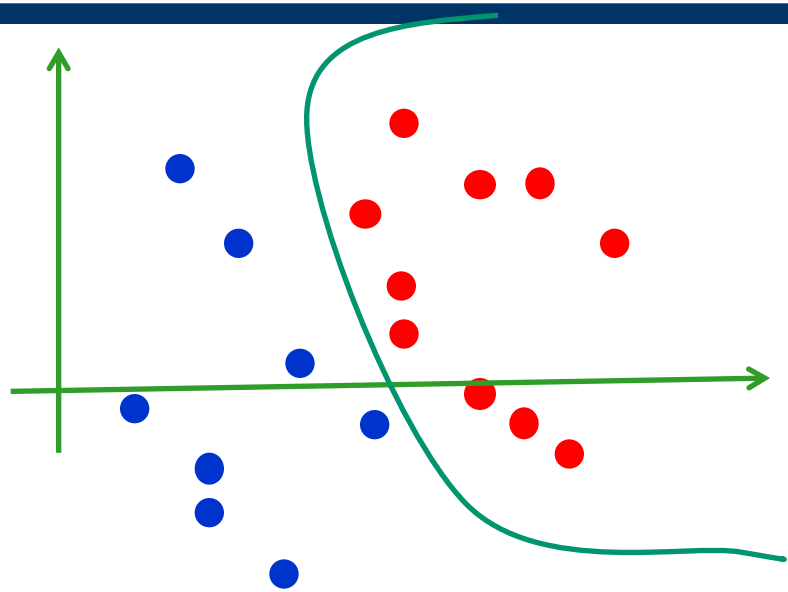
Determination of the linear separating border with Linear Discriminant Analisys (LDA). We assume that the two classes have a gaussian distribution with different means and identical covariance matrix

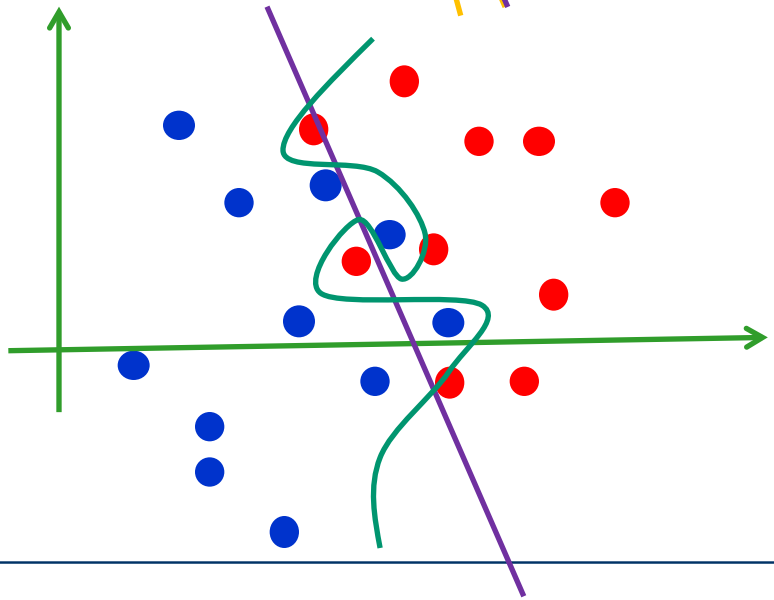
- Squares correspond to LDA values of real fractures
- Circles correspond to LDA values of false alarms



Some issues we need to focus on…

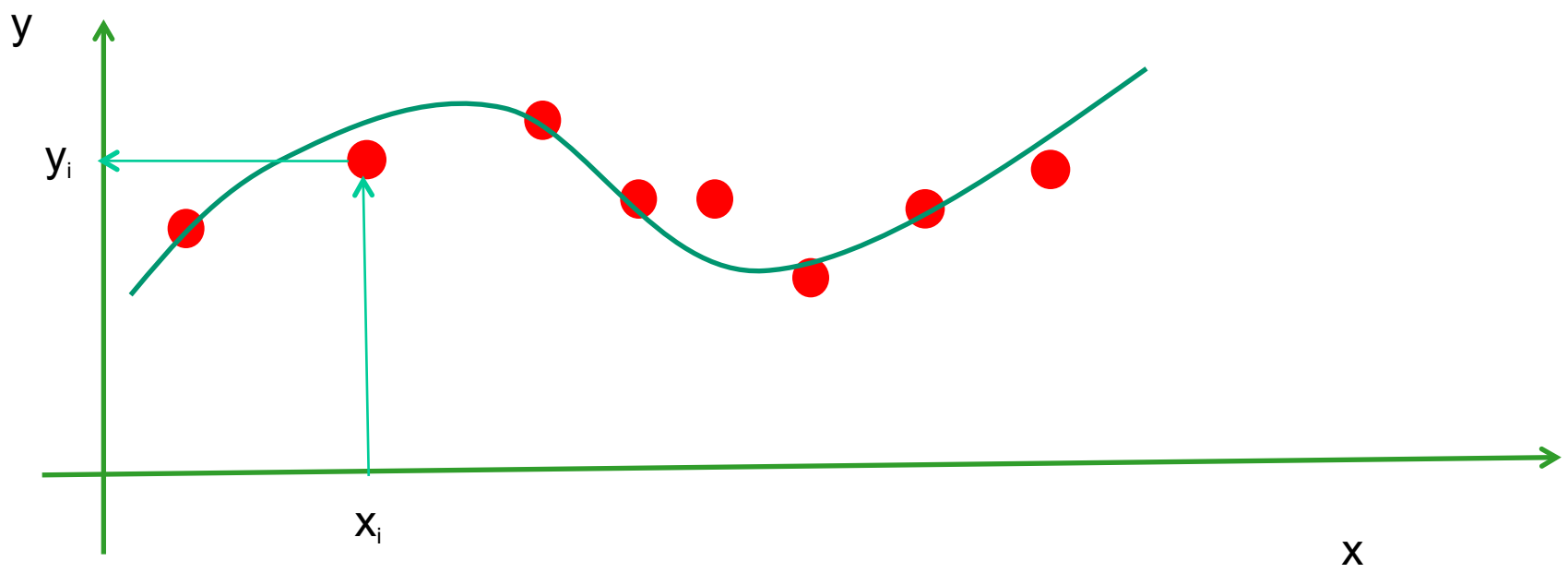






- Linear vs. non linear
- Many points versus the available points
- Several tecniques are available to design the classifier (KNN, feedforward, SVM…)

Non linear regression



Given a set of *n* noise affected couples (x_i,y_i) we wish to reconstruct the unknown function

- **Supervised Learning: Statistical framework**

Non-linear regression: statistical framework

The time invariant process generating the data

$$y = g(x) + \eta,$$

provides, given input x_i output instance

$$y_i = g(x_i) + \eta_i$$

We collect a set of couples (training set)

$$Z_N = \{(x_1, y_1), \ldots, (x_N, y_N)\}$$

And wish to model unknown $g(x)$ with parameterized family of models $f(\theta, x)$

The goal of **learning** is to build the simplest approximating model able to explain past data Z_N and future instances provided by the data generating process.

Always exploit available information

The parameterized family of models $f(\theta, x)$ takes advantage of a priori information about $g(x)$.

- belongs to a nested hierarchy of model families
- is mostly continuous and differentiable
- is chosen according to the a priori information we have about $g(x)$ (e.g., gray box modeling), model complexity (linear families) or universal function approximation ability.
- Not rarely it is linear
- It can also have dynamics (e.g. AR, ARX, recurrent networks)

The statistical learning framework

Structural risk

$$\bar{V}(\theta) = \int L(y, f(\theta, x))\, p_{x,y} dxy$$

$$\theta^o = \arg\min_{\theta \in \Theta} \bar{V}(\theta)$$

Empirical risk

$$V_N(\theta) = \frac{1}{N} \sum_{i=1}^{N} L(y_i, f(\theta, x_i))$$

$$\hat{\theta} = \arg\min_{\theta \in \Theta} V_N(\theta)$$

Learning procedure

$$\theta_{i+1} = \theta_i - \eta \frac{\partial V_N(\theta, Z_N)}{\partial \theta}|_{\theta_i}$$

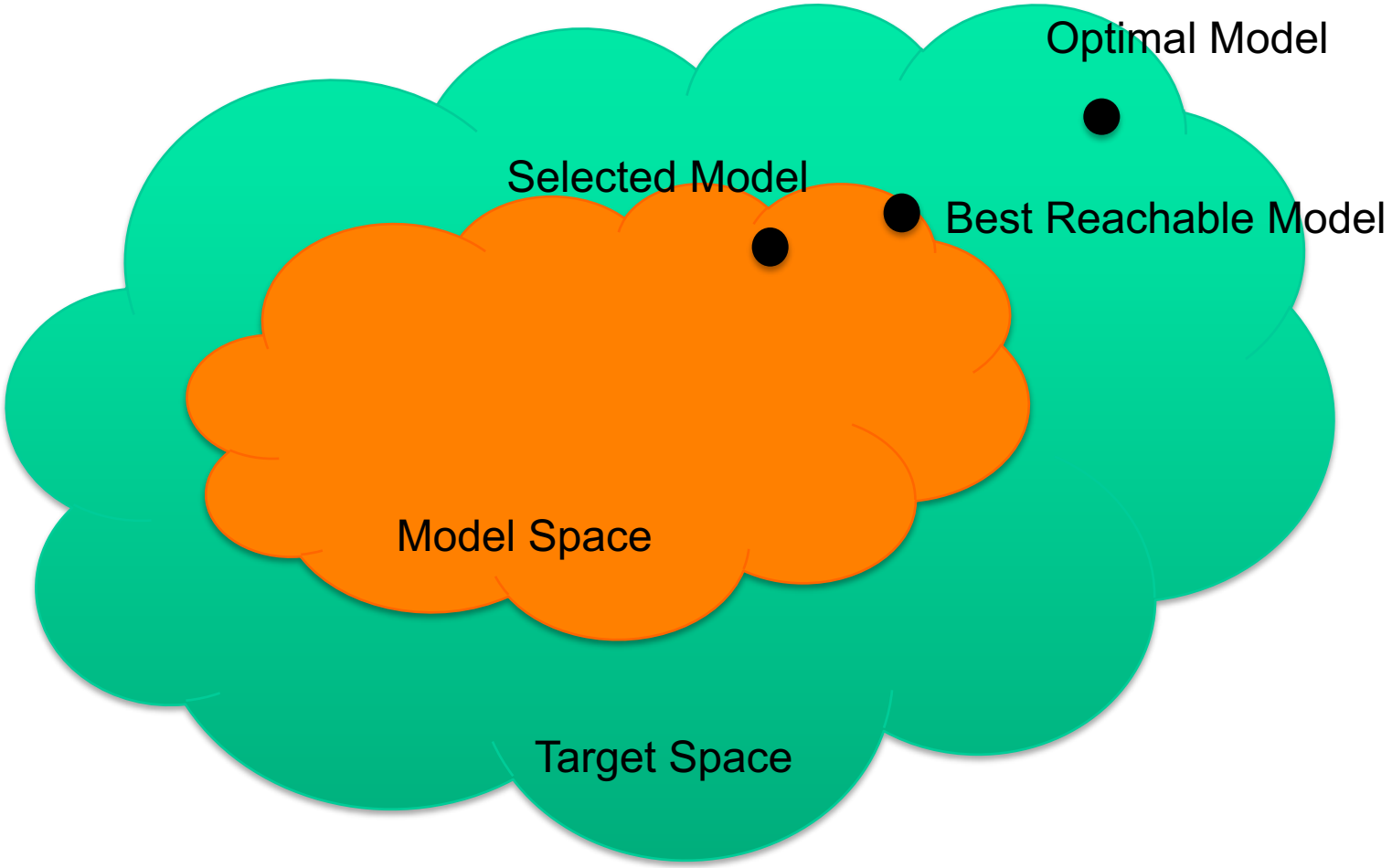
Inherent, approximation and estimation risks

$$\bar{V}(\hat{\theta}) = \left(\bar{V}(\hat{\theta}) - \bar{V}(\theta^o)\right) + \left(\bar{V}(\theta^o) - V_I\right) + V_I.$$

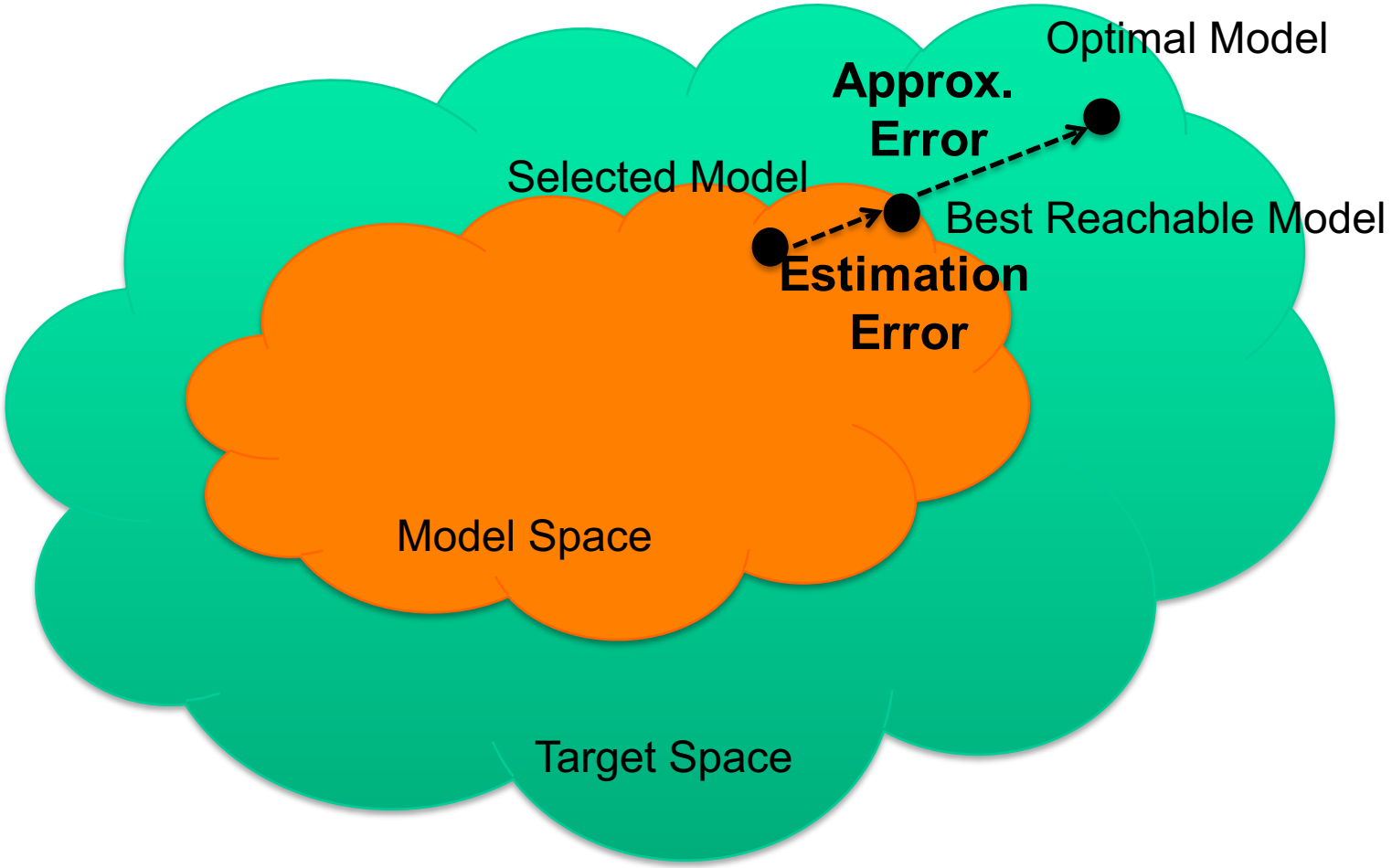
estimation risk | approximation risk | inherent risk

- The inherent risk depends only on the structure of the learning problem and, for this reason, can be reduced only by improving the problem itself
- The approximation risk depends on how close the model family (also named hypothesis space) is to the process generating the data
- The estimation risk depends on the ability of the learning algorithm to select a parameter vector $\hat{\theta}$ close to θ^o

Approximation and estimation risks

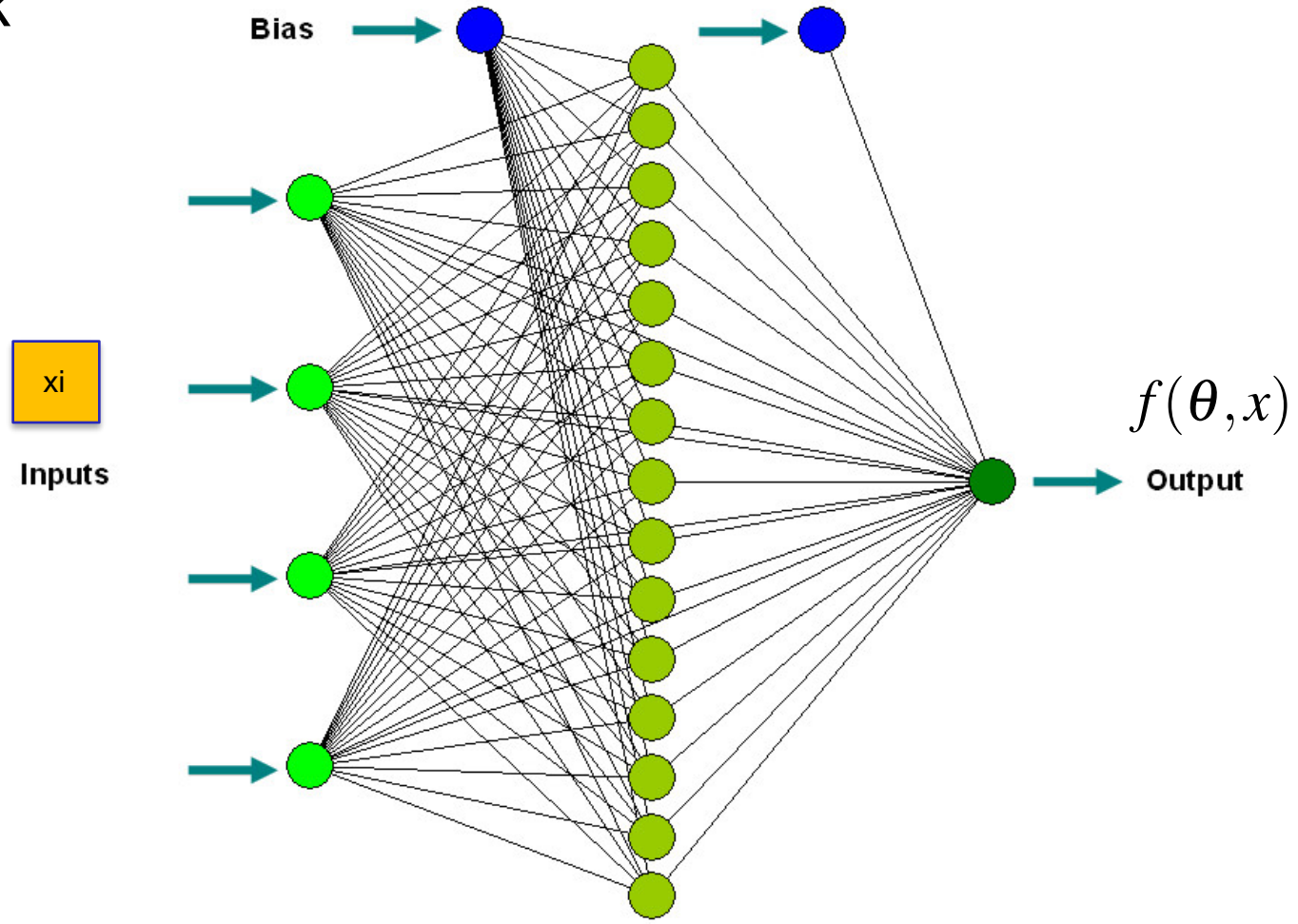


Approximation and estimation risks



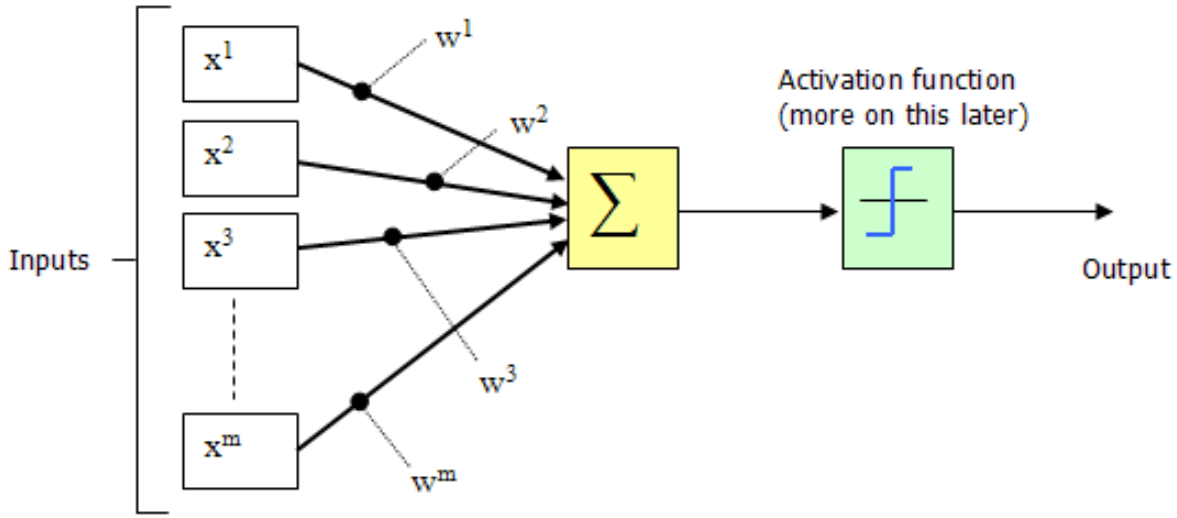
Choosing a universal function approximator

Not rarely $f(\theta, x)$ is chosen as a feedforward neural network



Choosing an universal function approximator

And the function implemented by a neuron is



The activation function can be

- A hard limiter function
- A sigmodal-like function
- A linear one (output layer)

Quality assessment of the solution

«How good is your ‘good’?»

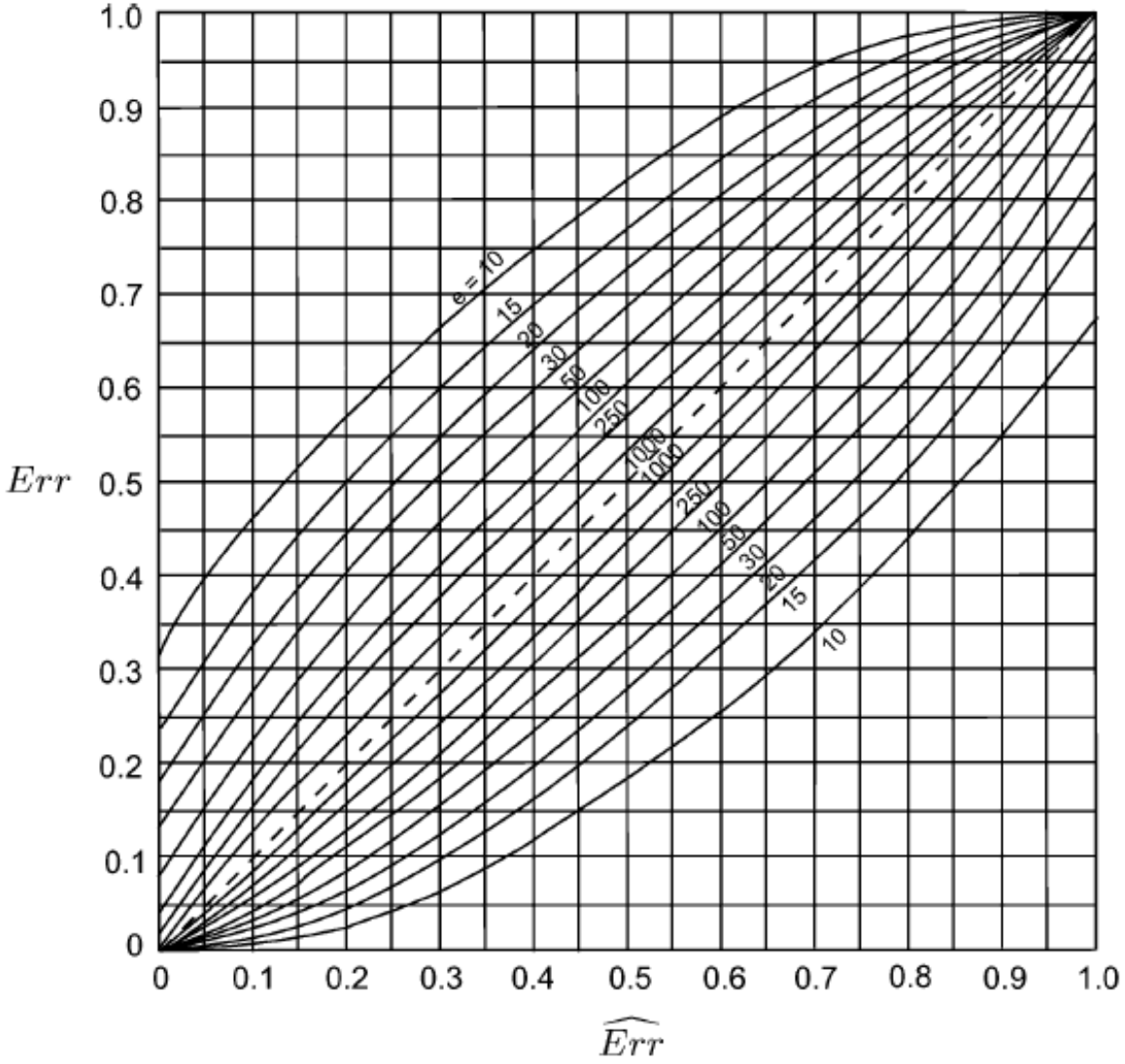
Assessing the performance (1)

- *Apparent Error Rate* (AER),or *resubstitution*: The whole set Z_N is used both to infer the model and to estimate its error
- *Sample Partitioning* (SP): S_D and S_E are obtained by randomly splitting Z_N in two disjoint subsets. S_D is used to estimate the model and S_E to estimate its accuracy.
- *Leaving-One-Out* (LOO): S_E contains one pattern in Z_N, and S_D contains the remaining $n - 1$ patterns. The procedure is iterated n times by holding out each pattern in Z_N, and the resulting n estimates are averaged.

Assessing the performance (2)

- w-*fold Crossvalidation* (wCV): Z_N is randomly split into w disjoint subsets of equal size. For each subset the re- maining $w - 1$ subsets are merged to form S_D and the reserved subset is used as S_E . The resulting w estimates are averaged. This procedure can be iterated and the results averaged when $w \ll n$ in order to reduce the random resampling variance. This estimate is a generalization of LOO.

An example: Binomial law confidence interval for the classifier accuracy



But ….

… what about the confidence of an estimator?

- **Bootstrap**
- **Bag of little bootstrap**

Quality assessment

Consider a data set Z_n obtained by extracting n i.i.d. samples $x_1, \ldots, x_n$ from random variable x defined over X, i.e., $Z_n = \{x_1 \ldots, x_n\}$ and construct the estimator $\Phi_n = \Phi(Z_n)$. We are interested in providing an indication of the quality ζ of Φ_n, e.g., we wish to provide a confidence interval for Φ_n.

Clearly, the ideal framework would recommend to carry out the following procedure

1. Extract m independent data sets of cardinality n from X so as to generate datasets $Z_n^1, \ldots, Z_n^m$;
2. Evaluate, in correspondence of the generic i-th data set Z_n^i the estimator $\Phi_n^i = \Phi(Z_n^i)$. Repeat this procedure for all $i = 1, \ldots, m$.;
3. Estimate the quality $\zeta\left(\Phi_n^1, \ldots, \Phi_n^m\right)$ of the estimator Φ_n based on the m realizations $\Phi_n^i = \Phi(Z_n^i), i = 1, \ldots, m$.

The bootstrap

Unfortunately, the above framework is mostly theoretical: if we have m independent datasets Z_n we should use all nm data to provide a better estimate. This means that in practical applications we have only a dataset but, at the same time, we are interested in evaluating the quality ζ of the estimator Φ_n.

- In the bootstrap method data sets $Z_n^i,\ i = 1, \ldots, m$ are extracted with replacement from Z_n

Algorithm 15: The bootstrap algorithm

```
i = 0;
while i < m do
    Extract n samples with replacement from Z_n and insert them in Z_n^i;
    Compute Φ_n^i = Φ(Z_n^i);
    i = i+1;
end
Evaluate the assessment ζ(Φ_n^1, ..., Φ_n^m) of the quality of the estimator Φ_n.
```

The bag of little bootstraps method

- BLB shows to be accurate and appears to over-perform all other bootstrap methods in terms of computational complexity, hence becoming a very appealing method for Big Data

Algorithm 16: The Bag of Little Bootstraps algorithm

$n' = n^{\gamma}$;
i = 0;
while $i < m$ **do**
  Extract n' samples **without replacement** from Z_n and insert them in $Z^i_{n'}$;
  j = 1;
  while $j < r$ **do**
    Extract n samples **with replacement** from $Z^i_{n'}$ and insert them in $Z^i_{n,j}$;
    Compute $\Phi^i_{n,j} = \Phi(Z^i_{n,j})$;
    j=j+1;
  end
  Evaluate $\zeta_i = \zeta\left(\Phi^i_{n,1}, \cdots, \Phi^i_{n,r}\right)$;
  i = i+1;
end
Evaluate the assessment ζ for of the quality of the estimator Φ_n as $\zeta = \frac{\sum_{i=1}^{m} \zeta_i}{m}$.

e.g., γ=0.6

Let's play with MATLAB

- Download the examples related to Lecture 3 – Part 2
- In the ZIP file:
  - Example 3_2_A.m
    - Linear dataset, linear model
  - Example 3_2_B.m
    - Linear dataset, nonlinear model
  - Example 3_2_C.m
    - Nonlinear dataset, linear model
  - Example 3_2_D.m
    - Nonlinear dataset, nonlinear model
  - Example 3_2_E.m
    - Different number of hidden units
  - Example 3_2_F.m
    - Classification and Apparent Error Rate
  - Example 3_2_G.m
    - Classification and Sample Partitioning